Whence

$$x = -\frac{r}{2} \pm \frac{r}{2}\sqrt{16n+1}.$$

Letting $r^2 = A^2$, $B = 8nr^3$, and $C = \sqrt{\frac{2B}{A^3}+1} - 1$, we have

(1) $$x = \frac{AC}{2}.$$

But

$$\frac{B}{\frac{A^3C^3}{8} + \frac{3A^3C}{2}} = 1.$$

Hence,

(2) $$x = \frac{AC}{2} \times \frac{B}{\frac{A^3C^3}{8} + \frac{3A^3C}{2}} = \frac{4B}{A^2(C^2+12)}.$$

From the last two values of x, we have, by addition and dividing by 2,

(3) $$x = \frac{2B}{A^2(C^2+12)} + \frac{AC}{4}.$$

(2) This always gives exact values when $n = \frac{1}{2}$.

As an application, take the cubic equation,

$$x^3 + 12x = 1{,}120.$$

Here $A = 2$, $B = 1{,}120$, and $C = \sqrt{281} - 1$. Hence, from (1), $x = 15.7$; from (2), $x = 8.4$, and from (3), $x = 10.03$. Thus (3) gives a very close approximation.

Remark by the Editors.

The proposer does not answer (3), (4), and (5). The deductions in his solution result from the confusion of letters representing different quantities and using them to represent the same quantity without drawing the necessary conclusion therefrom. Thus, in the expression $\frac{1}{3}\pi x^2 \times (3r - x)$, the r here is the radius of the sphere from which the segment is taken, whereas in $\frac{1}{2}\pi r^2 x + \frac{1}{6}\pi x^3$, the r is the radius of the base of the segment. If these two expressions are equated as they are in the solution above, it follows that $x = r$ and therefore $n = \frac{1}{2}$. It thus follows that when $n = \frac{1}{2}$ the formula gives correct values for cubics of the proposed form. This seems to us to answer the remaining questions in his problem.

387. Proposed by H. C. FEEMSTER, York College, Nebraska.

Sum the following series:

(1) $$\sum_{n=1}^{n=\infty} \frac{1}{8^{n+1}n!}; \quad (2) \sum_{n=1}^{n=\infty} \frac{1\cdot 3\cdot 5 \cdots 2n+1}{2^{2n+1}(2n-1)!}; \quad (3) \sum_{n=1}^{n=\alpha} \frac{1}{2^{2n}n!}.$$

SOLUTION BY J. A. BULLARD, Worcester Polytechnic Institute.

By Maclaurin's Theorem, we have

$$f(x) = \sum_{n=0}^{n=\infty} \frac{x^n}{n!} D^n f(0), \qquad (A)$$

where $D^n f(0) = [D^n f(x)]_{x=0}$, $0! = 1$ and $D^0 f(0) = f(0)$.

(1) Let $f(x) = e^x$. Then by (A),

$$\frac{e^x}{8} = \frac{1}{8}\sum_{n=0}^{n=\infty}\frac{x^n}{n!} = \frac{1}{8}\left\{1 + \sum_{n=1}^{n=\infty}\frac{x^n}{n!}\right\}. \tag{B}$$

Letting $x = \frac{1}{8}$, and reducing, we readily have

$$\frac{e^{\frac{1}{8}} - 1}{8} = \sum_{n=1}^{n=\infty}\frac{1}{8^{n+1}n!}.$$

(2) $$\sum_{n=1}^{n=\infty}\frac{1\cdot 3\cdot 5\cdots 2n+1}{2^{2n+1}(2n-1)!} = \sum_{n=1}^{n=\infty}\frac{(2n+1)!}{2^{3n+1}(2n-1)!\,n!}$$

$$= \sum_{n=1}^{n=\infty}\frac{2n+1}{2^{3n}(n-1)!} = \frac{1}{8}\sum_{n=1}^{n=\infty}\frac{2n+1}{8^{n-1}(n-1)!}.$$

Let $f(x) = (2x+3)e^x$. Then $D^0f(0) = 3$, $D'f(0) = 5, \cdots, D^nf(0) = 2n+3$. By equation (A), we have

$$\frac{(2x+3)e^x}{8} = \frac{1}{8}\sum_{n=0}^{n=\infty}\frac{x^n}{n!}(2n+3).$$

Letting $x = \frac{1}{8}$, we have

$$\frac{(\frac{1}{4}+3)e^{\frac{1}{8}}}{8} = \frac{1}{8}\sum_{n=0}^{n=\infty}\frac{2n+3}{8^n n!}.$$

Replacing n by $n-1$, we have

$$\frac{13e^{\frac{1}{8}}}{32} = \frac{1}{8}\sum_{n=1}^{n=\infty}\frac{2n+1}{8^{n-1}(n-1)!} = \sum_{n=1}^{n=\infty}\frac{1\cdot 3\cdot 5\cdots(2n+1)}{2^{2n+1}(2n-1)!}.$$

(3) Letting $x = \dfrac{1}{2^2}$ in equation (B) and multiplying by 8, we have

$$e^{\frac{1}{4}} - 1 = \sum_{n=1}^{n=\infty}\frac{1}{2^{2n}n!}.$$

Excellent solutions were received from A. M. Harding, R. M. Mathews, G. N. Bauer, H. L. Slobin, and S. Lefschetz.

Note.—The Proposer intended the sign in the denominator of (2) to be $+$ instead of $-$. With that change his answer for (2) is $\frac{1}{2} - \frac{1}{3}\sqrt{2}$. Editor.

388. Proposed by JOSEPH V. COLLINS, Stevens Point, Wis.

On a certain typewriter there is a double scale as follows:

80··· 70···60···50···40···30···20···10··· 0
1··· 5···10···15···20···25···30···35···40

It is used to locate headings in the middle of the page. To space a heading, one sets the machine at the right stop and with the spacer counts out the number of letters and spaces in the heading. To the reading on the 40 scale where the carriage stops is added the reading of the right stop on the same scale. This number is the one on which to set the carriage pointer on the 80 scale to begin the heading. Show by algebra that the method is correct.